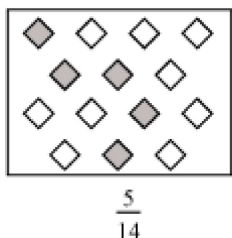
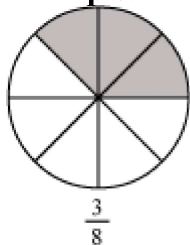


4. Operations on Fractions

- A fraction is a number representing a part of a whole.

The whole may be a single object or a group of objects.

Example:



- A **common fraction** is written in the form $\frac{a}{b}$, where a and b both are integers and $b \neq 0$. Here, a is numerator and b is denominator. Common fractions are also known as **vulgar** or **simple fractions**.
- While expressing a situation where parts have to be counted to write a fraction, it must be ensured that all parts are equal.

- For a fraction $\frac{2}{13}$, 2 is called its numerator and 13 is called its denominator.

Fractions are categorized into three types: proper, improper, and mixed fraction.

Proper fractions are those fractions in which the numerator is less than the denominator. These fractions are always less than 1.

For example, $\frac{17}{24}$ is a proper fraction since the numerator, 17, is less than the denominator, 24.

Improper fractions are those fractions in which the numerator is greater than the denominator. These fractions are always greater than 1.

For example, $\frac{15}{7}$ is an improper fraction since the numerator (15) > denominator (7).

A mixed fraction is a combination of a whole number and a part.

For example, $9\frac{5}{13}$ $\left[9\frac{5}{13} = 9 + \frac{5}{13}\right]$

If numerator and denominator of a fraction are equal, then the fraction is 1.

For example, $\frac{5}{5} = 1$

A mixed fraction can be converted into an improper fraction as
$$\frac{(\text{Whole} \times \text{Denominator}) + \text{Numerator}}{\text{Denominator}}$$

For example, $8\frac{2}{23} = \frac{(8 \times 23) + 2}{23} = \frac{184 + 2}{23} = \frac{186}{23}$

- To convert an improper fraction into a mixed fraction, first of all, the quotient and remainder are obtained by just dividing the numerator by the denominator. Then, the mixed fraction corresponding to the given improper fraction is written as

$$\text{Quotient} \frac{\text{Remainder}}{\text{Divisor (Denominator)}}$$

For example, to find the mixed fraction corresponding to the improper fraction $\frac{182}{7}$ first of all, 182 is divided by 17.

Here, divisor = 17, quotient = 10, and remainder = 12

$$\therefore \frac{182}{17} = 10\frac{12}{17}$$

- Addition of two like fractions can be performed just by adding the numerators and retaining the denominator of the fractions.

For example, $\frac{17}{25} + \frac{3}{25} = \frac{17+3}{25} = \frac{20}{25} = \frac{20 \div 5}{25 \div 5} = \frac{4}{5}$

- Subtraction of two like fractions can be performed just by subtracting the numerators and retaining the denominator of the fractions.

For example, $\frac{31}{15} - \frac{4}{15} = \frac{31-4}{15} = \frac{27}{15} = \frac{27 \div 3}{15 \div 3} = \frac{9}{5}$

- To perform the addition and subtraction of unlike fractions, first of all, they are converted into their equivalent fractions with the denominator as the LCM of their denominators. Then, addition or subtraction can be performed easily.

Example:

Find the sum of $\frac{4}{3}$ and $\frac{5}{12}$.

Solution:

LCM of 3 and 12 = 12

$$\therefore \frac{4}{3} + \frac{5}{12} = \frac{4 \times 4}{3 \times 4} + \frac{5 \times 1}{12 \times 1} = \frac{16}{12} + \frac{5}{12} = \frac{21}{12} = \frac{21 \div 3}{12 \div 3} = \frac{7}{4}$$

Example:

Subtract $\frac{4}{33}$ from $\frac{3}{22}$.

Solution:

LCM of 33 and 22 = 66

$$\therefore \frac{3}{22} - \frac{4}{33} = \frac{3 \times 3}{22 \times 3} - \frac{4 \times 2}{33 \times 2} = \frac{9}{66} - \frac{8}{66} = \frac{1}{11}$$

- To add or subtract mixed fractions, first of all, they are converted into improper fractions. Then, they can be added or subtracted easily.

For example,

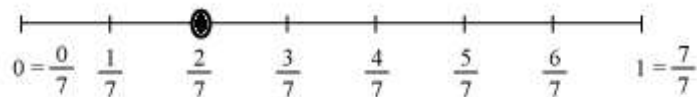
$$\begin{aligned} 7\frac{2}{5} + 3\frac{4}{9} &= \frac{37}{5} + \frac{31}{9} \\ &= \frac{37 \times 9}{5 \times 9} + \frac{31 \times 5}{9 \times 5} \quad (\text{LCM of 5 and 9 is 45}) \\ &= \frac{333}{45} + \frac{155}{45} \\ &= \frac{333+155}{45} \\ &= \frac{488}{45} \\ &= 10\frac{38}{45} \end{aligned}$$



- A fraction can be represented on a number line.

For example, to represent $\frac{2}{7}$ on the number line, first of all, the gap between 0 and 1 is divided into 7 equal parts. Then, each part represents the fraction $\frac{1}{7}$.

Therefore, $\frac{2}{7}$ can be easily represented on the number line as:



- **Multiplication of fractions with whole number**

- A whole number is multiplied with a proper or improper fraction by multiplying the whole number with the numerator of the fraction, keeping the denominator same. For example, $\frac{4}{3} \times 2 = \frac{8}{3}$
- A mixed fraction is first converted into an improper fraction and then multiplied with the whole number. For example, $1\frac{2}{3} \times 5 = \frac{5}{3} \times 5 = \frac{25}{3}$

Multiplication of fraction by fraction

When two fractions are multiplied, the product is obtained as $\frac{\text{Product of numerators}}{\text{Product of denominators}}$

For example, $\frac{2}{9} \times \frac{7}{3} = \frac{2 \times 7}{9 \times 3} = \frac{14}{27}$

The product of two proper fractions is always less than each of the fractions.

For example, $\frac{2}{3} \times \frac{4}{7} = \frac{8}{21}$ Here $\frac{8}{21} < \frac{2}{3}$ and $\frac{8}{21} < \frac{4}{7}$

The product of two improper fractions is greater than each of the fractions.

For example, $\frac{3}{2} \times \frac{7}{4} = \frac{21}{8}$ Here, $\frac{21}{8} > \frac{3}{2}$ and $\frac{21}{8} > \frac{7}{4}$

The product of a proper fraction and an improper fraction is greater than the proper fraction, but less than the improper fraction.

For example, $\frac{2}{3} \times \frac{7}{4} = \frac{2}{3} \times \frac{7}{4} = \frac{7}{6}$ Here, $\frac{7}{6} > \frac{2}{3}$ and $\frac{7}{6} < \frac{7}{4}$

- **Reciprocal of a number** is obtained by interchanging the numerator and denominator of that number.

For example, reciprocal of $\frac{1}{4}$ is $\frac{4}{1}$ or 4.

- **Division of fraction by whole number or fraction**

The fraction is multiplied with the reciprocal of the divisor. For example,

$$\frac{2}{9} \div \frac{4}{5} = \frac{2}{9} \times \frac{5}{4} = \frac{10}{36} = \frac{5}{18}$$

$$\frac{3}{11} \div 3 = \frac{3}{11} \times \frac{1}{3} = \frac{3}{33} = \frac{1}{11}$$

Division of whole number by fraction

The whole number is multiplied with the reciprocal of the fraction. For example, $2 \div \frac{1}{5} = 2 \times 5 = 10$

